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Key stages	KS1
School type	LA maintained, primary
Themes	Mathematics

## How valuable is the Concrete-Pictorial-Abstract model of teaching mathematics concepts across the primary school phase for all children, not just the Lower-Ability or SEN pupils?

### Maple Primary School

#### Context

Maple Primary School is an average- sized primary school located in the St Albans district of Hertfordshire.

#### The focus

On Friday 18<sup>th</sup> September 2015, the Herts for Learning maths team hosted a national conference with Jo Boaler, Professor of Mathematics at Stanford University, as the key note speaker. Many Hertfordshire teachers attended the conference to find out more about developing mathematical mindsets and were inspired to continue improving opportunities in mathematics for their pupils through an action research project. The purpose of the project was to explore some of the themes covered by Jo Boaler and research different ways of developing mathematical mindsets. *This case study has been written by Rachel de la Croix, Year2 Class Teacher and Maths Subject Leader at Maple Primary School.*

This focus was selected from the many aspects covered in Jo Boaler's presentation at the conference, linked to my own belief and research into the importance of visualisation (including the use of fingers!). When reading Boaler's 'Setting up Positive Norms in Maths Class', which is a list of her favourite messages to share about maths in the classroom. Two stood out: 'Maths is about creativity and making sense' and 'Maths is about connections and communicating'. These two areas really linked into my initial ideas of how we can show pupils how to make connections between maths concepts and communicate it in different ways.

I also have an interest in developing the 'Singapore bar method' for problem-solving in my school. I have had heard snippets about the method in courses over the past few years and the value of this model in a 'Mastery Curriculum'. This method to solving 'abstract problems' links closely to the concrete-pictorial-abstract (CPA) model. My decision was also based upon my personal values of what good maths teaching looks like: making it real; finding different ways of explaining the same thing; exploring 'hooks' for the children to grasp new things and the value of being very active in the use of visual and kinaesthetic methods of teaching.

Over the last 18 months, I have been actively integrating the use of Numicon to support learning in the classroom. The impact of these resources was having a positive effect, one which I wanted to extend further. Although the school's Year 2 maths results were above National and in-line with Hertfordshire figures, through analysis of Key Stage 1 data from May 2015, I questioned why the number of pupils achieving level 3 in maths was half that of the number of pupils achieving level 3 in reading and below the number of level 3s for writing. This 'gap' had not been seen in the Key Stage 2 results however. This led me to consider, as the maths subject leader and the Year 2 teacher: What are we doing differently for English that we aren't doing for maths? and What things can we do to

even things out? How could we make the teaching of maths more like the teaching of reading and writing through the use of the 'Read, Write Inc. scheme'?

## Description of my approach

The first part of the pro-active action research started with an anonymous teacher questionnaire, with the aim of finding out teacher's general knowledge and response to the concrete-pictorial-abstract (CPA) model of teaching. The response from staff was less than hoped for - only 50% returned. However, I gained several key points from the questionnaires returned:

- Some key manipulatives (place value equipment) were rarely being used in classrooms.
- Manipulative were kept in drawers in the classroom and either chosen by the class teacher or available if the pupil requested.

Teachers really valued the use of concrete and pictorial images but felt challenged by time and expectation to lead the children to more formal recording more quickly.

Examples of teachers' comments:

- "It's really useful to use as many as possible to help all different types of learners... Our TAs are really good at using them."
- "Children are good at using pictorial resources themselves- they are especially good at getting maths mats, whiteboards and number lines to support their learning."
- "They are vital to help children to 'see' the concept. They need to understand 'visually' before they can internalise the concept."

To help ascertain what current maths practice in the school looks like, I carried out a number of informal observations in the Foundation Stage, Year 1 and Year 2 classrooms during a number of maths lessons.

In the lesson observed, the class teachers were using a mixture of concrete and pictorial images to support the learners with symbols (numerals and calculation signs) used by the children who were ready for this. From talking to the Foundation Stage and Key Stage 1 teachers during the project, they can see the use of the CPA method of teaching as being something that needs to be adopted from the start of the children's learning and applied to all their learning experiences (not just maths!). There have been some valuable discussion about the use of our IWBs as a way of showing 'virtual resources' such as place value equipment and Numicon shapes. The correct organisation and modelling of the different manipulatives is also an area for development in the near future.

In the summer term, a book scrutiny was carried to see if there was evidence of pupils being encouraged to support their learning with pictorial representations. It showed that in Year 1 and Year 2 classes recording varied considerably. Children tended to be encouraged to show their pictorial representations on their whiteboards (photocopied for assessment after the lesson) or use manipulatives to support their learning but recording in books usually showed numbers and symbols for calculations.

In Key Stage 2 books, there is a small margin drawn where pupils can show their working - these tend to be more formal methods. Pictorial representations were shown rarely, usually linked to measurement or shape activities. Through staff training in the summer term, staff were encouraged to think about the purchase of the half squares/ half plain maths books to encourage the children's pictorial recording to be valued and used to assess understanding.

## Analysis of pupils' work

As a Year 2 class teacher, I was able to analyse whether pupils in my class had used their own pictorial representations to support them during the Key Stage 1 maths tests, where manipulatives of any sort weren't permitted. Since the start of the academic year, with the knowledge that whilst good teaching in maths included access to suitable supportive manipulatives, I realised that the absence of these in a test situation would be a huge disadvantage to many children. Therefore, I had purposefully integrated into my teaching, time to model how each calculation could be solved with a picture of some sort (where appropriate!)

I also carried out an audit of the maths resources in the Foundation Stage, Year 1 and Year 2 classes prior to my training session for all staff in these stages. I organised the audits into key areas of teaching (number and place value/ counting and calculations/ shape/ measures). These audits were then used as a starting point for discussion about what manipulatives we should be using to support learning in certain areas and how they can be organised for ease of access to pupils in the future. It would appear that whilst many teachers would like the children to be able to select their own manipulatives, this skill has to be developed over time, with key modelling by the class teacher being crucial to this process. Whilst we want to move away from only having resources in drawers, having a 'bounty' of resource on each working table can be equally 'off putting' and unsupportive to our learners! Key Stage 2 staff were asked to conduct a similar audit of the maths resources in their classrooms prior to staff and room changes in the Autumn term.

## Links with parents / home

Through this project, with the aim of supporting the teaching of maths as fully as we support the pupils' development of early literacy skills, I then focused on how the parents' of our pupils could be supporting their children more effectively.

A new school initiative, started in Autumn term 2016, was to send home a plastic zipped –bag of 'Practical Maths tools' (digit cards, dice, counters, number line, hundred squares etc.) to support homework activities. This has worked well during 2015/16. Therefore, this initiative will continue into 2016/17 and an opportunity for the Foundation Stage and Key Stage 1 teaching team to model the use of these to our pupil's parents will be integrated into our autumn term 'start of the year' meetings.

At the end of May, I planned and led staff training sessions (for Foundation Stage and Key Stage 1 teachers and teaching assistants) on the CPA model of teaching in Maths. The main aim of this training was to familiarise the staff with the value of the CPA model of teaching:

***...mathematical understanding is developed through using concrete, pictorial and abstract (or symbolic) representations. Children will travel along this continuum again and again, often revisiting previous stages when a concept is extended. Children use concrete objects to help them make sense of the concept or problem.... Whatever the objects are, they can be moved, grouped and rearranged to illustrate the problem. As the child's experience and confidence grows, they may no longer need physical objects to actually move around. Instead, they draw them... As understanding develops, children move on to use some form of abstract representation. This could be giving values to rectangular bars (bar model) to identify what is known and what is unknown, using a symbol to stand for a number or something else. It is important to realise that these are not stages gone through once, but a continuum. There will be occasions when a particular child will use concrete, pictorial and abstract representations all in one session.... (Taken from an extract from Cherri Mosely - Consultant and author for Rising stars mathematics).***

In summary, these are the key discussions that came from this staff training:

- The use of concrete manipulatives and pictorial representations need to be modelled well by the class teacher initially if the pupils are to be successful.
- Virtual resources are highly valuable - the maths subject leader has saved some to the 'staff resources'/ web links given too.
- Manipulatives need to be accessible and varied to support different styles of learners and different maths concepts.
- This model of teaching needs to be seen as a whole school approach - not just Key Stage 1
- *'Using the CPA approach, understanding is likely to be developed more quickly where children are encouraged to start from modelling a problem with concrete objects, before moving on to pictorial and abstract representations. Therefore, it is suggested that a variety of manipulatives should be made available in the classroom, and **not just for Key Stage 1 children**' (Mosely).*
- Parents are key to supporting our pupils- but they need to know how to use the different resources (Modelling is key again). Meetings planned for the autumn term,
- The Singapore bar model of problem solving is of interest to staff. An Inset day has therefore been planned for this for the whole staff in Autumn term 2016.

In order to find out what the children really think helps them in maths, HfL Primary Teaching and Learning Adviser (mathematics), Gillian Shearsby-Fox, visited my school to conduct research with small groups of pupils in the Foundation Stage, Year 1 and Year 2 classes using an approach taken from a piece of research carried out by Borthwick. Four pupils from each class were asked to draw a picture to show what helps them in maths. There were a number of key points observed from this research:

- Most of the children found it tricky to articulate the names of the manipulatives in the classroom and in some cases where it was kept in the classroom
- A few children drew abstract picture of a hundred square or number line; none drew a picture of their teacher or teaching assistant
- The Year 2 children seemed more keen to show a picture of themselves to show they did it in their heads (no help from equipment or picture was needed).

As a result of this research, I now can see that a structured approach to familiarising all pupils (not just lower ability or SEN groups) in all maths lesson with useful manipulatives needs to be followed. Further staff training needs to support the idea that maths equipment can and should be used not only to support learning but as a means of the children proving their calculations and understanding.

During the Summer Term a 'mastery' staff meeting was led by two other members of staff - this reiterated many of the key themes of the staff training I had previously led with the Key Stage 1 staff. A number of important discussion points were raised including the use of 'challenge activities' which the children choose and signals a move away from our traditional ability based groups for maths lesson. Staff also looked at what helped pupils become 'masters' at concepts - leading us to talk more about the CPA approach to teaching and learning.

## Pupil Voice

In the summer term, I talked to three pupils from each class, Year 1 to Year 6, and asked a number of questions about their use of, familiarity with and their attitude to the use of manipulatives and drawings to support their learning in maths. There seemed to be a number of key opinions shared by most pupils:

- Manipulatives are provided by the class teacher on tables to support 'less able' pupils.
- Manipulatives are kept in drawers and cupboards – the children knew where they were but rarely asked for them.

- Whilst the teachers modelled pictures for answering questions, the children liked to show that they could do things in their head (this was seen as a strength) rather than using ‘things’ to help them; and manipulatives are used to support learning rather than a means of proving their answers.

## Widening the approach

I decided to focus my work and research initially with the Foundation Stage and Key Stage 1 staff as I felt they were perhaps the most familiar with the CPA model already and could be a good advocate of this with the Key Stage 2 staff.

- The focus and initial impact of this study will be shared with school’s Governors in the autumn term.
- The approach will be widened further with Key Stage 2 staff in the autumn term.

## Impact and recommendations

### Changes in myself

I would say that I have definitely changed my approach overtime by asking myself, ‘How effective do I want this action research to be?’

Whilst the starting and finishing point would always be the pupils and what they could gain by our knowledge and skills as teachers, I needed everybody to be ‘on-board’ with supporting the children. Therefore, I wanted to start by gaining the support of my Foundation Stage/ Key Stage 1 team and then widen it to include the Key Stage 2 team and parents. At all points, I kept in mind that I didn’t want to ‘confuse’ the children, but support them in understanding the key concepts within maths.

I have also seen the benefit of familiarising the children with using a range manipulatives and pictures to show their mathematical understanding.

Through my research, I am now in a better position to disseminate training throughout the school as to how the CPA approach is fundamental to achieving mastery in maths.

### Changes in the mind-sets of learners

In my class (Year 2), I saw a real change in my pupils’ openness to share their methods of calculation; sometimes with manipulatives, sometimes pictures, sometimes symbols; over the course of the project. Even the least confident pupils were happy to hold up their ‘whiteboards’ showing their methods, which were perhaps different to other children’s, but showed their way of being successful. By using praise for the varied methods of pictorial representation, the children were happy to show ‘their way’! And, as I have found out, the power of holding up a pupil’s method is unbeatable.

I often told the children, “...don’t struggle with a question, think... ‘What can I do or draw, to help myself?’” From looking at the Key Stage 1 SATs maths papers and their regular maths books, I could see many of them doing this regularly.

During maths lesson, I was able to ask them ‘What could help us to do this?’ with great and varied response. The children began to ask for other manipulatives which they thought might help them and, with support, they were able to ‘prove it’ with manipulatives when asked to.

### Changes in teachers' attitudes

As the project has grown over the year, I have observed a number of behaviour / attitude changes in the other KS1 teachers at my school:

- The teachers are talking about 'useful' maths equipment far more. With a 'whole-school' classroom switch about to take place, the teachers are keen to ensure that they have adequate access for all children each lesson to a variety of key equipment.
- Teachers are coming to me for ideas about what the CPA model would look like for teaching particular maths concepts as they have seen how effective this model of teaching can be.

### Thoughts for next time

The teacher questionnaire, whilst some-what useful, didn't give the 'whole picture', due to the poor response. Next time, I would alter my starting point of research by using a short video clip of CPA in action and include all staff in an open discussion afterwards or give teachers time to complete a questionnaire at a staff meeting.

I would have chosen to have a Key Stage 2 'buddy' to work alongside during the project, so that positive effects of the knowledge gained through the research could be spread more quickly across the school.

### Future actions (2016/2017)

- FS and KS1 staff to familiarise parents with the names of and value of 'Maths Manipulatives' and 'Pictorial Representations' to support learning in maths in their start of year meeting.
- KS2 Audit of maths equipment - how is it organised, modelled and used by teachers and all pupils?
- Whole staff Training: Developing the CPA model across the school - use of the Singapore bar method. (Autumn Term 2016)
- All teachers will be encouraged to model the use of manipulatives in all maths lessons so that children are confident with the names of and uses of them.
- Maths lessons must take place every day (Like 'RWInc' lesson) to ensure the same attitude to maths.
- All teachers to be encouraged to think of manipulatives as a way of pupils proving their maths rather than just supporting it. Papert (1980) calls manipulatives "*objects to think with.*"

***"Incorporating manipulatives into mathematics lessons in meaningful ways helps students grasp concepts with greater ease, making teaching most effective".***

<b>Contact</b>	Rachel de la Croix, Year 2 Class Teacher and Maths Subject Leader at Maple Primary School
<b>Reading and website references</b>	<p>School website: <a href="http://maple.herts.sch.uk/">http://maple.herts.sch.uk/</a></p> <p>Leong Yew Hoong, Ho Weng Kin, Cheng Lu Pien : Concrete-Pictorial-Abstract: Surveying its origins and charting its future</p> <p>Nrich The Role of Mastery in Nurturing Young Mathematicians</p> <p>Black.J. (2013) Manipulatives in the Primary Classroom -Stage: 1 and 2</p> <p>Boaler.J . Positive Norms to encourage in Maths classes</p> <p>Moseley.C. What-is-the-Concrete-Pictorial-Abstract-(CPA)-approach?</p> <p>Borthwick.A. Children’s perceptions of, and attitude towards, their mathematics lessons.</p> <p>Boaler.J. :Activities for finger training</p> <p>Handtomind.com: Research on the Benefits of Manipulatives</p> <p><i>Websites:</i>  <a href="http://www.risingstars-uk.com">http://www.risingstars-uk.com</a>  <a href="http://nrich.maths.org">http://nrich.maths.org</a>  <a href="http://www.youcubed.org">www.youcubed.org</a></p>

If you have an aspect of interesting practice that could be shared or are interested in finding out more about a case study please get in touch by emailing [exchangingexcellence@hertsforlearning.co.uk](mailto:exchangingexcellence@hertsforlearning.co.uk)

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