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Billy Counts

The Fluency Project

Herts for Learning

Dated	July 2015
Key stages	KS1
Themes	Mathematics

Introduction

Over the last year Herts for Learning (HfL) has been engaging with schools to explore how pupils' number sense can be improved. Working alongside the research community of teachers and schools, HfL advisors have uncovered some uncomfortable truths about how schools may be stifling pupils' innate sense of number rather than fostering it. Teachers from the community were asked to identify groups within their class they felt could be designated as 'high calculators' (confident calculators), 'mid calculators' (somewhat confident) and 'low calculators' (not confident calculators). Pupils' ability to subitise, use appropriate strategy to calculate and estimate relative size of number (number magnitude) were assessed.

The following article by Rachel Rayner and Charlie Harber (Primary Mathematics Advisers from Herts for Learning Ltd.) sets out some of the findings from their ongoing research and work with schools.

Context

Billy was given the calculation 5+6 to solve. He counted out five fingers and held them up, he seemed pleased and smiled. Then he looked at the calculation and counted out six more fingers. This confused Billy. He needed one more finger. So he began again, in the same way, and hit the same obstacle. He wanted to provide an answer. So he looked at the last group of fingers he put up and said, confidently, "six". Through all of the questions he was asked, Billy used only one strategy - to count all. He was only accurate when he could calculate using the counting environment of his fingers.

Billy is typical of the children we meet on our travels. He was in year 1 and receiving SEND provision, when we met him, in a good school with a great teacher. But the lone strategy he relied on was a common feature amongst many other children we see. Imagine reaching year 3 or 4 and having to use fingers when calculating in the context of formal written methods for addition and subtraction. But this is what we see in our day to day work in schools. Children, from a range of designated ability groups and a variety of school contexts, who have not mastered the basics and, as a result, struggle with the added cognitive load. These children do not have number sense, why?

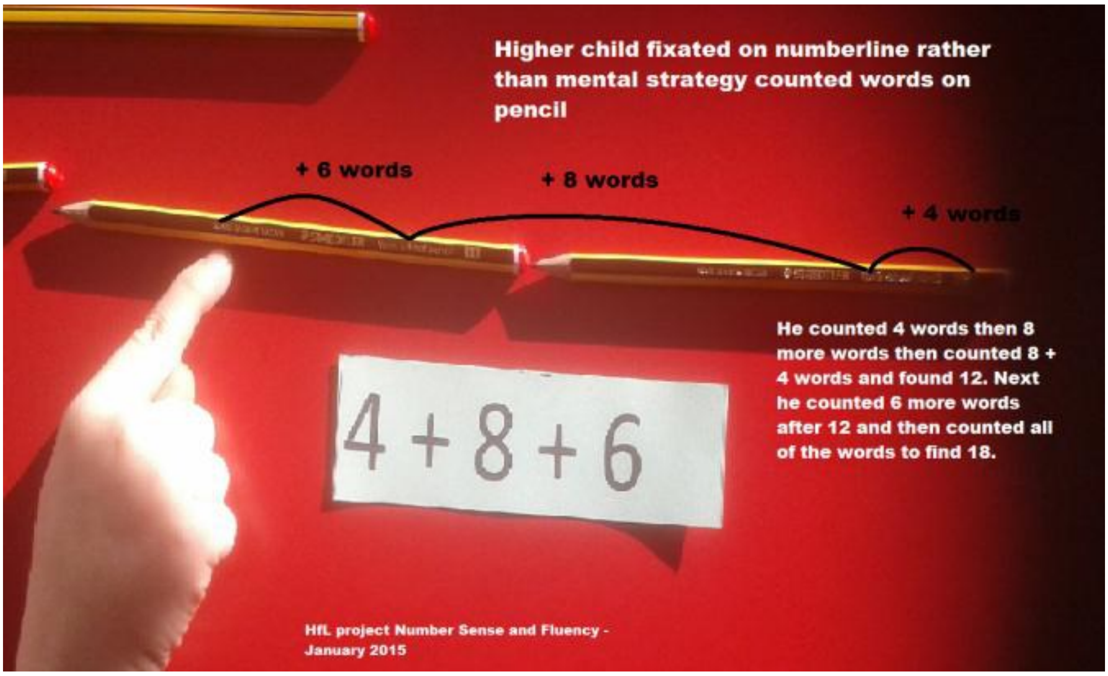


Figure 1 Tom's strategy

Another child, Tom, was designated as a ‘high calculator’ by his year 2 teacher. He was thought to be accurate in class and confident when calculating. Tom was given the calculation 4 + 8 + 6 to solve. His method turned out to be similar to Billy’s. He counted all too. But for the calculations in which he didn’t have enough fingers, his approach became a little more innovative. Tom took some pencils, he found on the table, and laid them end to end to create a number line. “I’m going to count the words on the pencils, like a number line,” he explained. Tom proceeded to count four words and placed his finger there. Then he counted eight more words and placed his finger at that point. Next, he counted the four words and the eight words to find the total so far. He continued to count the remaining six words, placed his finger as a marker. He then counted from the first word to where his finger now marked the total. He provided his answer “17” then checked and said “18”.

The third of our children was Flora, again identified by her year 2 teacher as a high calculator, who when assessed revealed counting to be her primary strategy. But Flora was quick. You’d never know she was counting at all; just a flicker of her fingers gave her away. She had some really great methods for counting too, for example: partitioning numbers, counting the ones, then the tens and recombining. Her teacher was shocked to find her lack of strategy; she had been identified as a high calculator because she was fast. Other children, deemed as displaying less ability as calculators, actually had far more strategies in their repertoire. The full range of demonstrated strategies are coded for ease of data analysis and explained within figure 2.

Count all	**CA**	Counts 5 then counts 6 then counts all
Counts on	**CO**	Conserves 5 then counts on 6 more (or counts back for subtraction)
Make 5	**M5**	Uses knowledge of 5 to calculate e.g. 2 + 3 or 8 - 3
Make 10	**M10**	Uses knowledge of facts to 10 to calculate e.g. 4 + 8 + 6
Near make 10	**NM10**	Uses knowledge of facts to 10 and adjusts e.g. 8 + 5 becomes (8 + 2) + 3
Doubles	**D**	Uses knowledge of doubles e.g. 3 + 3 or 4 + 2 + 4 or 12-6
Near doubles	**ND**	Uses knowledge of doubles and adjusts e.g. 3 + 4 or 12 - 7
Commutativity	**CM**	Reorders calculations for ease e.g. 4 + 7 + 4 = 4 + 4 + 7

Figure 2 Strategies assessed

The first of the pie charts overleaf (top left) shows the ideal range of strategies that could be selected by year 2 pupils in an entry assessment. For each calculation assessed researchers identified the range of efficient strategies that could supplant counting and tracked pupil’s selections through the data. The remaining charts demonstrate that ‘count all’ (the darker blue) and ‘count on’ (burgundy) were used as strategies for the majority of the time by ‘mid’ and ‘low’ groups. Children designated as ‘low calculators’ used counting for over 75% of the time. We discovered a lot of reliance upon counting where teachers least expected to find it, namely in the group designated as ‘high’ calculators. This was replicated in year 1, except that the ‘count all’ strategy had a higher presence especially noticeable in the group of children designated as ‘low calculators’. Furthermore, counting made pupils less accurate in calculating as it was more likely to result in error. Not for Flora though. She was an incredibly fast and accurate counter. But could she continue as the number ranges she met increased to hundreds, thousands and beyond or would she start to feel the cognitive load increase?

Pupils in every group had strategies that they were developing, which can be seen from the pie chart. They could use a ‘doubling known facts’ to solve calculations such as 5 + 7 + 5 and also reason that 5 + 6 must be eleven because 5 + 5 is ten and then add one more (a near double strategy). There were core facts that most of the pupils could recall such as double five. However, in terms of fact recall and partitioning number, many of the children were insecure.

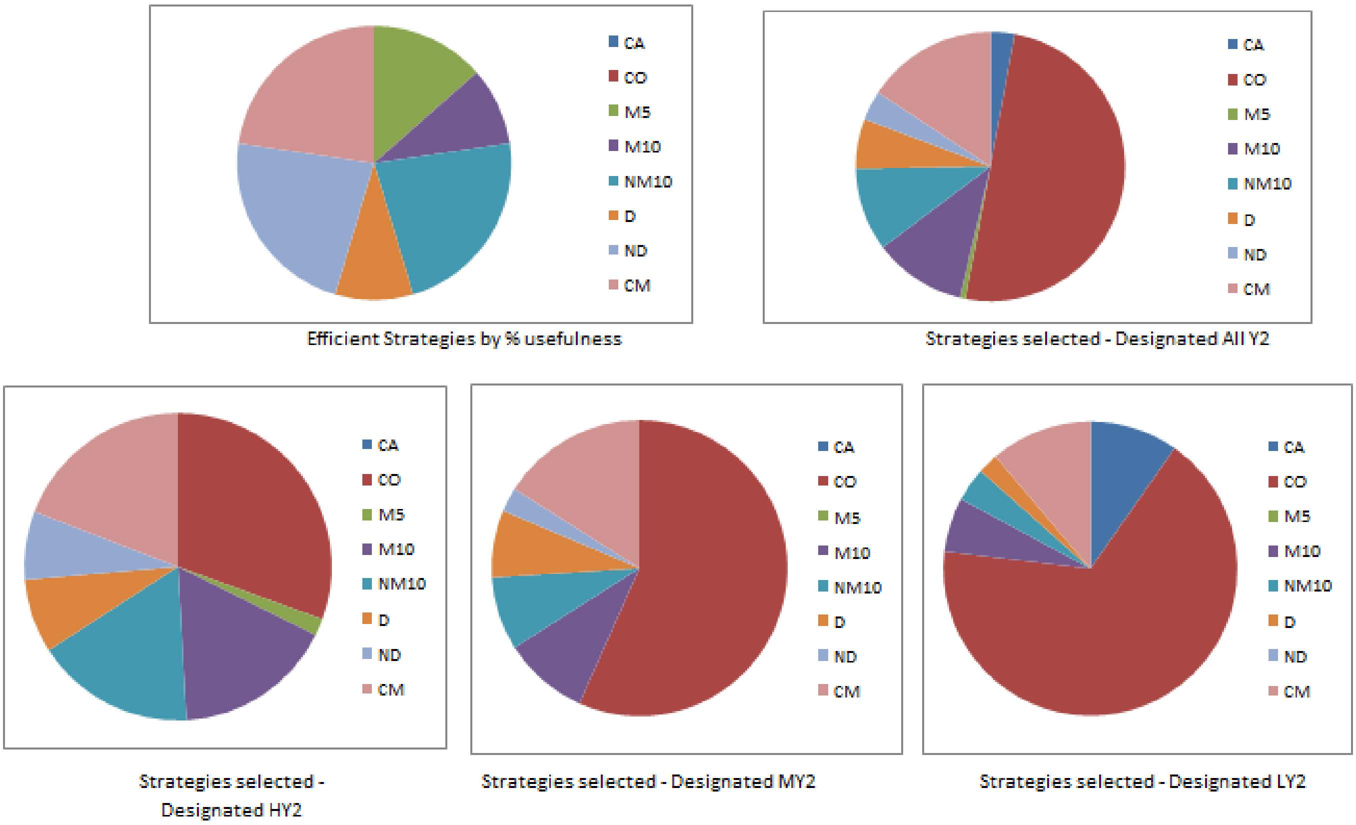


Figure 3 Research entry data for year 2

In our own research reading, we had identified that pupils' development in conceptual subitisation and in estimating the relative size of numbers impacted directly on their number sense.

Would children with limited strategies also demonstrate weaknesses in these areas?

Subitising

Have a look at the dot pattern – what do you see? Do you see seven dots? And how did you know there were seven dots?

Many pupils were not able to see the groups of dots (e.g. 2, 2 or 4 and 3 more) and combine them to find the total. They counted the dots instead. Seven, it transpires, is the hardest of all numbers to conceptually subitise. Billy and Flora were very weak in this area and although they could both see a group of 2 dots or 3 dots without counting, when assessed at baseline, they had no conceptual subitisation. We began to predict who the counters would be from the subitisation data. Those pupils with the ability to combine the dots or see *numbers within number* had greater fact recall and strategy to call upon when calculating. But Tom was great at subitising and this provided us with a conundrum. Tom had different barriers which we will explore later in this report.

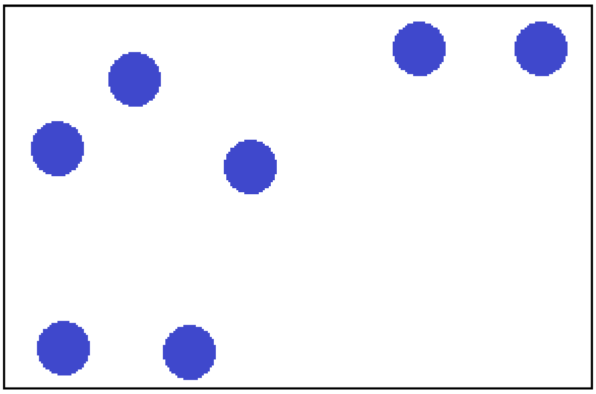


Figure 4 Conceptual subitisation - the ability to see groups of dots and to recombine to say the whole number

Number magnitude

We estimate number every day, for example looking at baskets of shopping in the queues at the supermarket or on the conveyor belt to decide which checkout queue to join. Our natural sense of number estimation develops as we get older. Research by Siegler and Booth (2004) showed that pupils' ability to accurately plot numbers, on a 0 – 100 number line, improves with age. They found that 'the linearity of children's number line estimates correlates positively with their existing knowledge of addition'.
We discovered that, on entry, the vast majority of our children presented an immature understanding of number magnitude. Flora and Billy over-estimated the smaller numbers when placing them on the number line and did not acknowledge key landmark points, such as half way, to support number placement, and often 'ran out' of space for the larger numbers. But Tom was much more reasonable in his estimation bucking the overall trend. Figure 5 shows how the children became more accurate in their placement of number, on a unlabelled 0 to 100 number line, shifting from a logaritihimic to a much more linear profile. Their understanding of the spatial relationships between numbers (0 to 100) improved significantly.

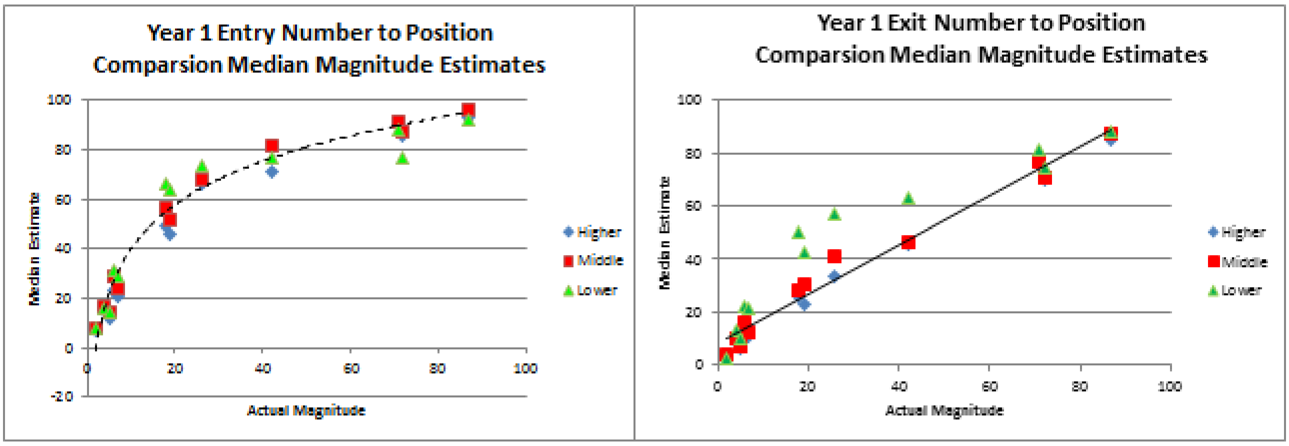
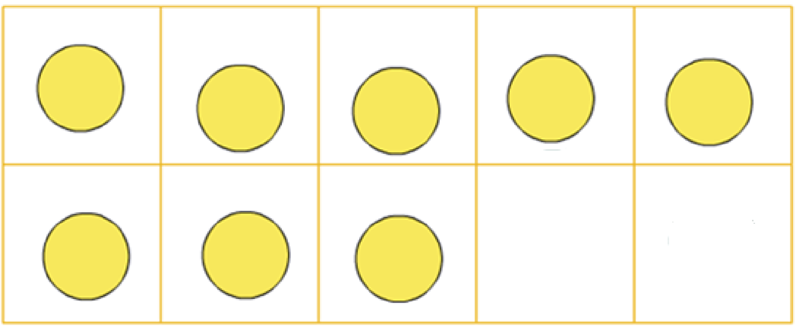


Figure 5 Median number line estimates and best fitting function based on Siegler and Booth (2004)

Adopting new approaches

We were privileged to work with an amazingly innovative community of teachers on this six week research project. After the baseline data collection, they admitted shock at what this had uncovered. But it spurred them on to improve outcomes for the pupils. We spent a day training them in the approaches devised, focussing on each of the three areas that we felt could improve: namely subitising, number magnitude and calculation strategy.



Ten frames

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Our primary manipulative was the **tens frame**, as we needed something effective that could increase capacity for conceptual subitisation *and* guide towards efficient strategies. Here we have the story of eight. Pupils were asked to tell us all the combinations that they can see to make 8. Some attended to the pairs, some to the double 3 and 2 more, some saw the 2 empty spaces to notice that 8 is 2 less than 10. We challenged them to be unique: who can find one that no one has thought of yet? Teachers modelled the symbolic notation for each combination and guided pupils into strategy. Pupils were provided with tens frames and asked to make 7 for example. They were then asked to adjust the frame to show 4 and tell a partner what they did alongside speaking frames which supported the talk e.g. “I had _ and I took away _. I have _ left.” Teachers did this frequently before progressing onto two, tens frames. Talk focussed upon the strategies and teachers set up strategy boards such as the one below to celebrate the diverse strategies pupils were using. The tens frame was a flexible image. Counters could be rearranged to reveal something different about the number 8 for example. A double-sided counter could be flipped or moved and a new combination exposed. It also provided the anchor points of 5 and 10 that pupils needed to use. The number seven (that trickiest of single digit numbers), for example, became ‘*2 more than 5*’ or ‘*3 less than 10’* which reinforced for pupils the sense of its relative size. The tens frame was successfully supporting the linking of conceptual subitisation to the ability to calculate and develop fact recall and strategy.

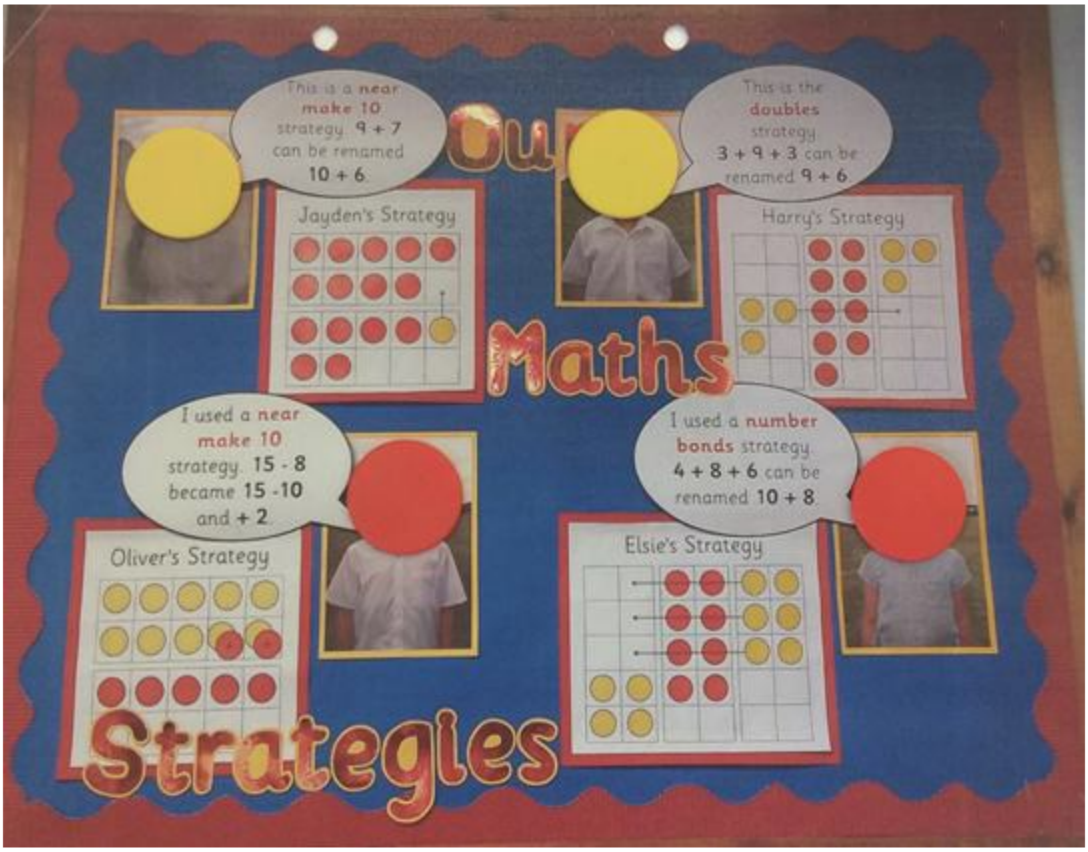


Figure 6 KS1 Strategy board celebrating pupils methods

The pupils became more confident with these and we observed great enthusiasm during these number-talk sessions. Teachers reflected, at a cluster meeting mid-way through the project, that children began to understand that in any number lived other numbers - these could be used to help. They described really inspiring moments. For example, when a child would come up with a really inventive strategy, partitioning and conserving numbers with confidence and the teachers were amazed at the range of strategy and depth pupils were reaching with relatively small numbers.

What of number magnitude? Plenty of research had identified that a weakness in number magnitude at five years old predicted a deficit in mathematical ability aged thirteen. But there seemed to be a lack of consensus or research about whether we could change these outcomes and, most importantly, how to do it. This meant we had to be very creative and, as a community, develop our own new approaches.

In the pilot project, one teacher worked very successfully with 'estimation zones' to encourage pupils to order using the landmark numbers. She named the quarter way landmark as 'the key' and the three quarter way landmark as 'the lock'. By the end of the six weeks of the pilot project, her pupils were observed by researchers adeptly generalising that if the three quarter mark (lock) was 12 then the other points must be 0, 4, 8, and 16. That enabled them to place the number the teacher had set, which was 6, very accurately. Another teacher placed a star on strips of paper and simply asked her pupils to say, "the number could be because… it couldn't be because…" Again, the level of conversation her six and seven year old pupils were having was impressive. We fed these ideas into the next stage of the project. The picture below demonstrates how one teacher used the idea of estimation zones but also used paper clips to show that when there were a lot of numbers in each zone they would be closer together and when there were fewer numbers they would be further apart – essentially the idea of intervals of scale.

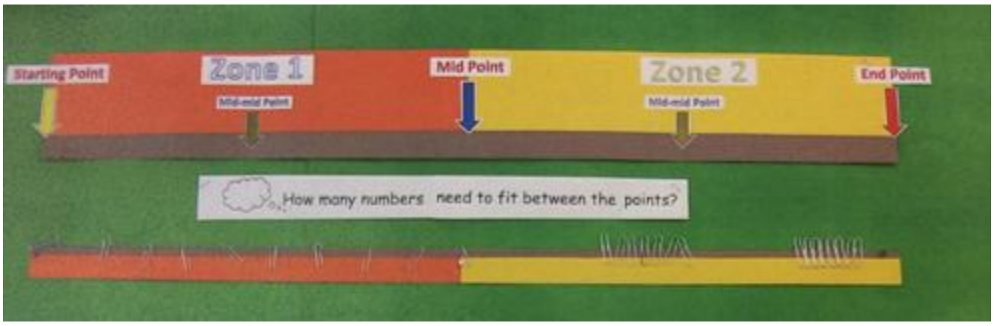


Figure 7

Number zones and intervals of scale in a mixed year 1 and 2 class

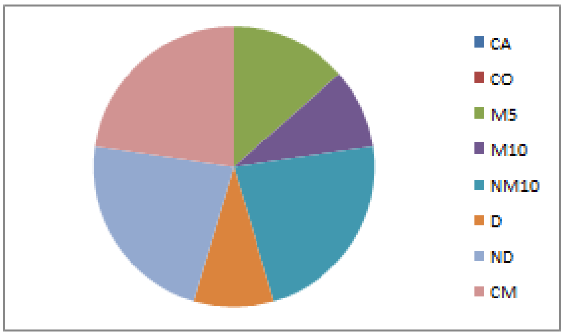
Outcomes of the project

In only six weeks, from baseline to exit assessment, the children made significant progress. Comparing strategies on entry to those used by pupils at the end of the project (see pie charts in figure 3 'entry' and figure 8 'end' below), it is clear that the reliance on counting (darker blue and burgundy) has decreased dramatically.

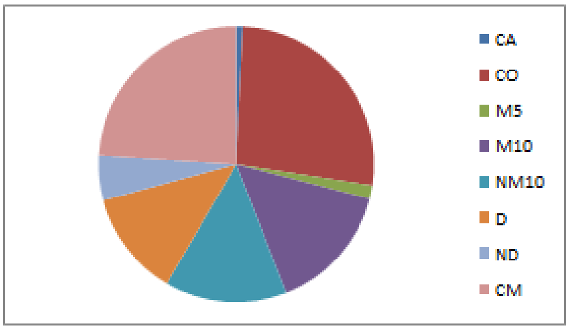
Commutativity or 'switching it', as we called in the project, is more secure and the use of other, more effective strategies are far closer to the proportions in our 'ideal' pie chart (top left).

Billy, now answers 5 + 6 using his near make ten rule: partitioning the 6 into 5 and 1 more. He uses four efficient strategies: near make ten and make five; commutativity (switching it) and doubles (and recalls his double facts). He has become more accurate answering six of the eight questions correctly. He puts up 6 fingers straight away without counting them demonstrating his increasing ability to conceptually subitise. Although he still struggles with his number magnitude, he is developing a more linear profile.

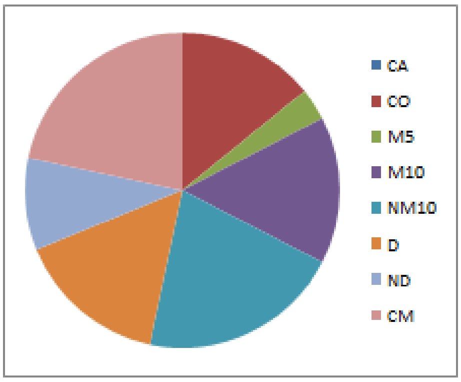
For Flora, counting is no longer her primary approach. She demonstrated a full range of appropriate strategies and is partitioning numbers well to 'bridge through landmark numbers'. She's not yet quite as quick as she used to be when counting but this will improve as one of the things we have learned is automaticity (calculating within 3 seconds) develops last. Crucially, her conceptual subitisation increased and her number magnitude results became linear, ahead of age expectations.



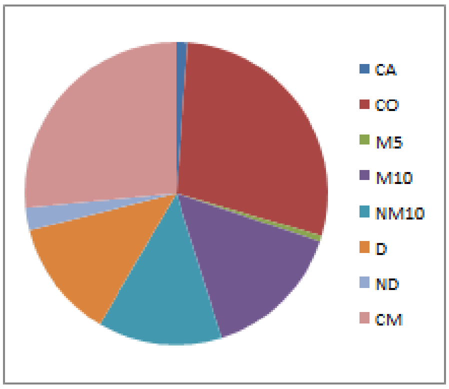
Efficient Strategies by % usefulness



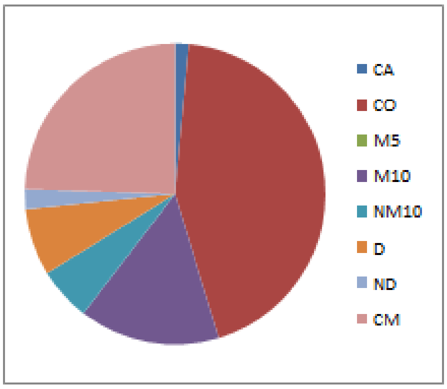
Strategies selected - Designated All Y2



Strategies selected -



Strategies selected - Designated MY2



Strategies selected - Designated LY2

Figure 8 Strategy Exit data

For Tom, it transpired that because his class used the number line as a representation frequently, this has encouraged him to use the 'count on and back' strategy. We have found this is commonly the case. Where the number line is the primary representation, pupils are pushed to develop counting as a strategy, often counting in ones rather than partitioning. We would argue that whilst the labelled number line helps pupils conceptually see the four operations, it does not help develop their recall of facts and appropriate use or secure number sense and these should be developed in tandem.

Our conclusions

Tom, Flora and Billy are real children who exemplify the traits that we commonly observed and tracked through the data. We should consider the importance of spending longer partitioning and exploring single digit numbers with our pupils. Speeding through this stage causes gaps to occur. Even where strategy is explicitly taught, pupils without partitioning skills are unlikely to be able to access them. Anchoring numbers to five and then ten is crucial. Some pupils are being accelerated too quickly beyond that. Moving into abstract representations too soon also halts pupils' development in number sense.

Fundamentally, if we do not explore and encourage the ability to conceptually subitise then this will lead to insecure strategy. But as this research community demonstrates, through carefully selected approaches, we can impact greatly on our pupils' numerosity.

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